Note on Boppart and Krusell (2020, Section V.B)

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February 3, 2026

1 Derivation of the GHH quasi-linear utility from the parametric class

Equation (8) in Section V.B (Parametric Forms) specializes to the Greenwood, Hercowitz, and Huffman (1988) quasi-linear utility function reported in equation (9).

Start from the general parametric class:

$$u(c,h) = \frac{c^{1-\sigma}\left[1 - a\left(hc^{\frac{\nu}{1-\nu}}\right)^{b}\right]^{d} - 1}{1-\sigma}. \tag{8}$$

Define the composite term

$$x := hc^{\frac{\nu}{1-\nu}}.$$

The goal is to obtain the GHH quasi-linear utility

$$u(c,h) = \frac{\left[c - \frac{w}{1-\frac{1}{\nu}}\, h^{1-\frac{1}{\nu}}\right]^{1-\sigma} - 1}{1-\sigma}. \tag{9}$$

Hence we must make the numerator of (8) equal to

$$c - \frac{w}{1-\frac{1}{\nu}}\, h^{1-\frac{1}{\nu}}$$

In (8), the term $c^{1-\sigma}$ is multiplied by $[1 - ax^{b}]^{d}$. To combine them into a single power of the form $[\,\cdot\,]^{1-\sigma}$, it suffices to set

$$d = 1 - \sigma,$$

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since then

$$c^{1-\sigma}[1-ax^b]^{1-\sigma} = \left[c(1-ax^b)\right]^{1-\sigma}.$$

This matches the exponent $(1-\sigma)$ in (9).

With $d = 1-\sigma$, the inside term becomes

$$c(1-ax^b) = c - a\,c\,x^b.$$

To match (9), the second term must be proportional to $h^{1-\frac{1}{\nu}}$. Compute $c\,x^b$:

$$c\,x^b = c\left(hc^{\frac{\nu}{1-\nu}}\right)^b = c\,h^b\,c^{\frac{\nu}{1-\nu}b} = h^b\,c^{1+\frac{\nu}{1-\nu}b}.$$

The exponent on c must be zero:

$$1 + \frac{\nu}{1-\nu}b = 0 \quad\Longrightarrow\quad b = \frac{\nu-1}{\nu} = 1 - \frac{1}{\nu}.$$

With this choice,

$$c\,x^{1-\frac{1}{\nu}} = h^{1-\frac{1}{\nu}}.$$

With $d = 1-\sigma$ and $b = 1-\frac{1}{\nu}$, we have

$$c(1-ax^b) = c - a\,h^{1-\frac{1}{\nu}}.$$

In (9), the term inside the brackets is

$$c - \frac{w}{1-\frac{1}{\nu}}\,h^{1-\frac{1}{\nu}}.$$

Therefore, a must equal the coefficient $\frac{w}{1-\frac{1}{\nu}}$.

In fact, after setting $d = 1-\sigma$ and $b = 1-\frac{1}{\nu}$, the inside term simply becomes

$$c(1-ax^b) = c - ah^{1-\frac{1}{\nu}}.$$

Comparing this with the inside of (9), we require

$$c - ah^{1-\frac{1}{\nu}} = c - \frac{w}{1-\frac{1}{\nu}}h^{1-\frac{1}{\nu}}.$$

This holds if and only if

$$a = \frac{w}{1-\frac{1}{\nu}}.$$

Summary

To obtain the GHH quasi-linear utility function, impose the following parameter restrictions:

$$\boxed{d = 1 - \sigma, \quad b = 1 - \frac{1}{\nu}, \quad a = \frac{w}{1 - \frac{1}{\nu}}.}$$

Substituting these into (8) yields

$$\begin{aligned} u(c,h) &= \frac{c^{1-\sigma}\left[1 - \frac{w}{1-\frac{1}{\nu}} x^{1-\frac{1}{\nu}}\right]^{1-\sigma} - 1}{1-\sigma} \\ &= \frac{\left[c\left(1 - \frac{w}{1-\frac{1}{\nu}} x^{1-\frac{1}{\nu}}\right)\right]^{1-\sigma} - 1}{1-\sigma} \\ &= \frac{\left[c - \frac{w}{1-\frac{1}{\nu}} h^{1-\frac{1}{\nu}}\right]^{1-\sigma} - 1}{1-\sigma}, \end{aligned}$$

which coincides with (9).

Finally, for the logarithmic case $\sigma = 1$, taking the limit yields

$$\lim_{\sigma \to 1} \frac{z^{1-\sigma} - 1}{1 - \sigma} = \log z,$$

so that

$$u(c,h) = \log\left(c - \frac{w}{1 - \frac{1}{\nu}} h^{1-\frac{1}{\nu}}\right).$$

References

Boppart, T. and P. Krusell (2020). Labor supply in the past, present, and future: A balanced-growth perspective. *Journal of Political Economy 128*(1), 118–157.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review 78*(3), 402–417.